Optimization technique for selecting minimum number of Secondary Schools in the network of rural habitation centers.

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Abstract: An interesting method is developed here to know the minimum number of Secondary Schools which are to be newly built up in some regions of rural habitations; and also to select the appropriate locations within the network of habitation centers of certain rural region of our country. This is done in such a way that every habitation center is covered by at least one these Secondary Schools within a distance of six kilometers. Here, the connected Graph problem is solved by using simple concept of game theory.

Key words: Optimization Technique, Game Theory, Graph Theory, Minimization Problem, Adjacency Matrix

1: Introduction

Education in our country is provided by the public sector as well as by the private sector, with control and funding coming from three levels: Central Government, State Governments and local authority. Under various articles of the Indian Constitution, free and compulsory education is provided as a fundamental right to children between the ages of 6 and 14. But it is a huge task for the education department who are responsible for implementing 'Sarva Shiksha Abhiyan' **(In English,** The Education for All Movement**).**

By a primary school, we mean a school where children are taught upto class IV or class V. An Upper primary (middle) school is a school having highest class upto VII or VIII. A Secondary School is a School having highest class upto IX or X.

In our country, Secondary education serves as a link between the elementary and higher Education; and occupy a very important position in the education system. A child's future can depend a lot on the type of education she/he receives at the secondary level. Apart from grounding the roots of education of a child, secondary education can be instrumental in shaping and directing the child to a bright future. This stage of education serves to move on higher secondary stage as well as to provide generic competencies that cut across various domains of knowledge as well as skills.

In this paper we are interested to identify appropriate locations of Secondary Schools within some specified region of rural habitations using mathematical techniques. We define the distance of a Secondary School from a rural habitation by the distance between the central point of the rural habitation and the School. A village may have more than one rural habitation center.

To select the appropriate locations of the Secondary Schools within a specified region and optimize the number of Schools, we consider the following assumptions:

a) At least one of the rural habitations lies within the radius of six kilometers, say, from the location of a Secondary School and;

b) Every rural habitation is covered within a distance of six kilometer by at least one of the Secondary Schools.

On the other way, we may say that a villager has to move at most six kilometer from his/her residence for a Secondary School. It is an optimization problem and it becomes necessary to introduce optimization techniques for the solution of such problem. We think of a mathematical optimization model for minimizing the number of Schools for the region.

In this paper, we consider a small model problem and it can be generalized in the similar way for larger problems. As an example, we consider that there are eleven rural habitations centered at the points A **to** K **in some rural region of our country. These centers are connected by road network as shown in** Figure-I **and distances between the habitations centers are given in kilometer.**

Here, we have to find out the minimum number of Secondary Schools and their prime locations in the region in such a way that every student can reach to a Secondary School within a distance of **six kilometers**. This type of connected and weighted Graph problem is generally solved by the method of minimum spinning tree. Kruscal and Prim algorithms are well known methods for finding spanning trees. Other methods (**Ref. [1] to [4]**] are also available in the literature. But when the network is large and weight of edges also large numbers, above methods may create problem for finding solution using personal computer. Here, we are going to introduce a new but simple method by which we can solve such basic problems of our country.

2: Solution Method

An interesting method is introduced here for the solution of the above problem using two persons zero sum game model as used in the Game Theory (**Ref [5] to [8]).** The distance (in kilometer) between population centers, are shown in the adjacency matrix in **Table-I**.

In this method we replace the distance between two habitation centers by the value one, if the distance between them is less than or equal to six kilometers; and replace the distance by value zero, if the distance is greater than six kilometer or when two habitation centers are not directly connected by a road. This method not only gives us the minimum number of Secondary Schools but also some important information about the locations of the Secondary Schools within the region.

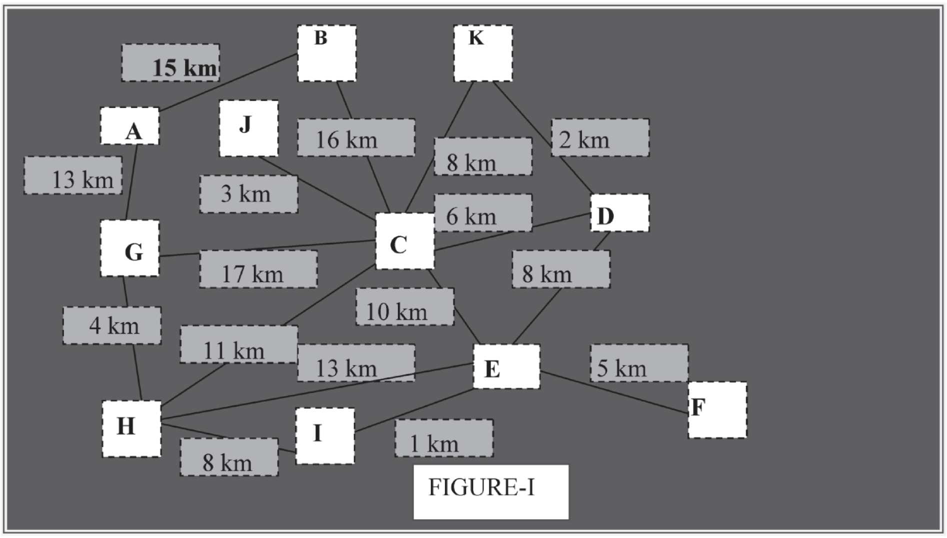

FIGURE-I

The newly created adjacency matrix by zeros and ones of the given problem is shown in **Table-II**. We may now think that this adjacency matrix as the pay-off matrix **(Ref. {5] to [8])** for the player **X** (shown at the left of the Table-II) and the second player **Y** is shown at the top of the **Table-II**.

	A	B	C	D	E	F	G	H	I	J	K
A	0	15	-	-	-	-	13	-	-	-	-
B	15	0	16	-	-	-	-	-	-	-	-
C	-	16	0	6	10	-	17	11	-	3	8
D	-	-	6	0	8	-	-	-	-	-	2
E	-	-	10	8	0	5	-	13	1	-	-
F	-	-	-	-	5	0	-	-	-	-	-
G	13	-	17	-	-	-	0	4	-	-	-
H	-	-	11	-	13	-	4	0	8	-	-
I	-	-	-	-	1	-	-	8	0	-	-
J	-	-	3	-	-	-	-	-	-	0	-
K	-	-	8	2	-	-	-	-	-	-	0

TABLE-I: Adjacency Matrix. All the dashes indicate that there no direct connection between the habitation centers.

PLAYER: Y →

PLAYER: X ↓

	A	B	C	D	E	F	G	H	I	J	K
A	1	0	0	0	0	0	0	0	0	0	0
B	0	1	0	0	0	0	0	0	0	0	0
C	0	0	1	1	0	0	0	0	0	1	0
D	0	0	1	1	0	0	0	0	0	0	1
E	0	0	0	0	1	1	0	0	1	0	0
F	0	0	0	0	1	1	0	0	0	0	0
G	0	0	0	0	0	0	1	1	0	0	0
H	0	0	0	0	0	0	1	1	0	0	0
I	0	0	0	0	1	0	0	0	1	0	0
J	0	0	1	0	0	0	0	0	0	1	0
K	0	0	0	1	0	0	0	0	0	0	1

TABLE-II: Adjacency matrix is converted in the binary matrix.

Using the rule of row dominance as used in the Game theory for two-person zero sum game, remove rows corresponding to **F, H, I, J** and **K** respectively and the game reduces to a game as given in **Table-III**.

PLAYER: Y →

PLAYER: X ↓

	A	B	C	D	E	F	G	H	I	J	K
A	1	0	0	0	0	0	0	0	0	0	0
B	0	1	0	0	0	0	0	0	0	0	0
C	0	0	1	1	0	0	0	0	0	1	0
D	0	0	1	1	0	0	0	0	0	0	1
E	0	0	0	0	1	1	0	0	1	0	0
G	0	0	0	0	0	0	1	1	0	0	0

TABLE-III: Using dominance, we remove rows corresponding to F, H, I, J and K.

Using column dominance (as done in the Game Theory), remove columns **C**, **D, E, H and I** respectively**.** After removing above columns, the pay off matrix reduces to a game which is given in **Table-IV**.

PLAYER: Y →

PLAYER: X ↓

	A	B	F	G	J	K
A	1	0	0	0	0	0
B	0	1	0	0	0	0
C	0	0	0	0	1	0
D	0	0	0	0	0	1
E	0	0	1	0	0	0
G	0	0	0	1	0	0

TABLE-IV: Using dominance, we remove columns C, D, E, H and I.

Rearranging columns of Table-IV with the diagonal elements one, we get Table- V.

3: Result and Discussion:

Table-V is the irreducible form of the game. We cannot reduce the matrix anymore in smaller order. Order of the matrix, so obtained, gives us the minimum number of Secondary Schools for the region. For the above example with eleven habitation (population) centers connected by the network of roads (as shown in FIGURE-I), only **six** Secondary Schools are required at different locations within the region so that every student can be accommodated in a Secondary School which is within a distance of six kilometers from his/her residence. For selection of locations of the Schools, we have alternative solutions for the problem.

	PLAYER: Y →					
		A	**B**	**J**	**K**	**F**	**G**
PLAYER: X ↓	**A**	1	0	0	0	0	0
	B	0	1	0	0	0	0
	C	0	0	1	0	0	0
	D	0	0	0	1	0	0
	E	0	0	0	0	1	0
	G	0	0	0	0	0	1

TABLE-V: Irreducible form of the pay off matrix.

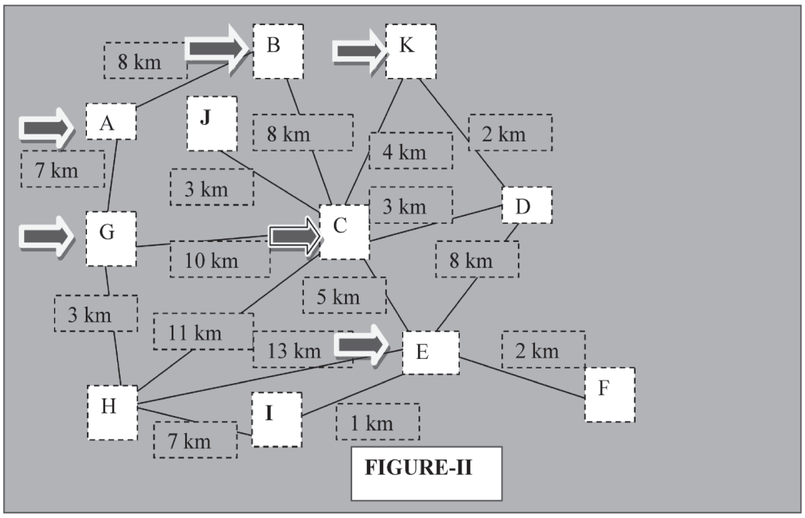

FIGURE-II

It can be seen from **Table-V** that six sub-graphs are formed, namely, **{A}, {B}, {C, J}, {D, K}, {E, F} and {G}** and other population centers **I** and **H,** can be added at least in one of these sub-graphs. For large problems this can be done by searching method. Population center **I** becomes a new member of sub-graph **{E, F}** and population center **H** becomes a new member of the sub-group {G}. So, newly created sub-graphs are **{A}, {B}, {C, J}, {D, K}, {E, F, I} and {G, H}.** If a connected sub-graph, so obtained, is large then its center may be considered as the appropriate location for the Secondary School for that sub-graph. In **Figure-II**, six appropriate locations of the Schools are shown by arrow symbols and these are habitation centers **A, B, C**, **E**, **G and K.**

4: Conclusion

Two persons zero-sum game is introduced here to solve minimization type connected and weighted graph problem. Novelty of this method is that it is not only gives us minimum number of the Secondary Schools but also suitable locations of the Schools. In the example, we have also some freedom for alternative choice. We can replace center **G** by center **H** and center **E** by center **F**. This method may also be applied for selecting suitable locations of Primary Schools, Health Centers in the rural area, locations of deep tube wells and their minimum numbers optimally.

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